Homework #4

1) f(A,B,C,D)   if 000/111 z=1 else z=0   <u>SOP</u>

ABCD	Z	
0000	1	m_0
0001	1	m_1
0010	0	M_2
0011	0	M_3
0100	0	M_4
0101	0	M_5
0110	0	M_6
0111	1	m_7
1000	1	m_8
1001	0	M_9
1010	0	M_{10}
1011	0	M_{11}
1100	0	M_{12}
1101	0	M_{13}
1110	1	m_{14}
1111	1	m_{15}

$\Sigma m(0,1,7,8,14,15)$

$\Pi M(2,3,4,5,6,9,10,11,12,13)$

$m_0 + m_1 + m_7 + m_8 + m_{14} + m_{15}$

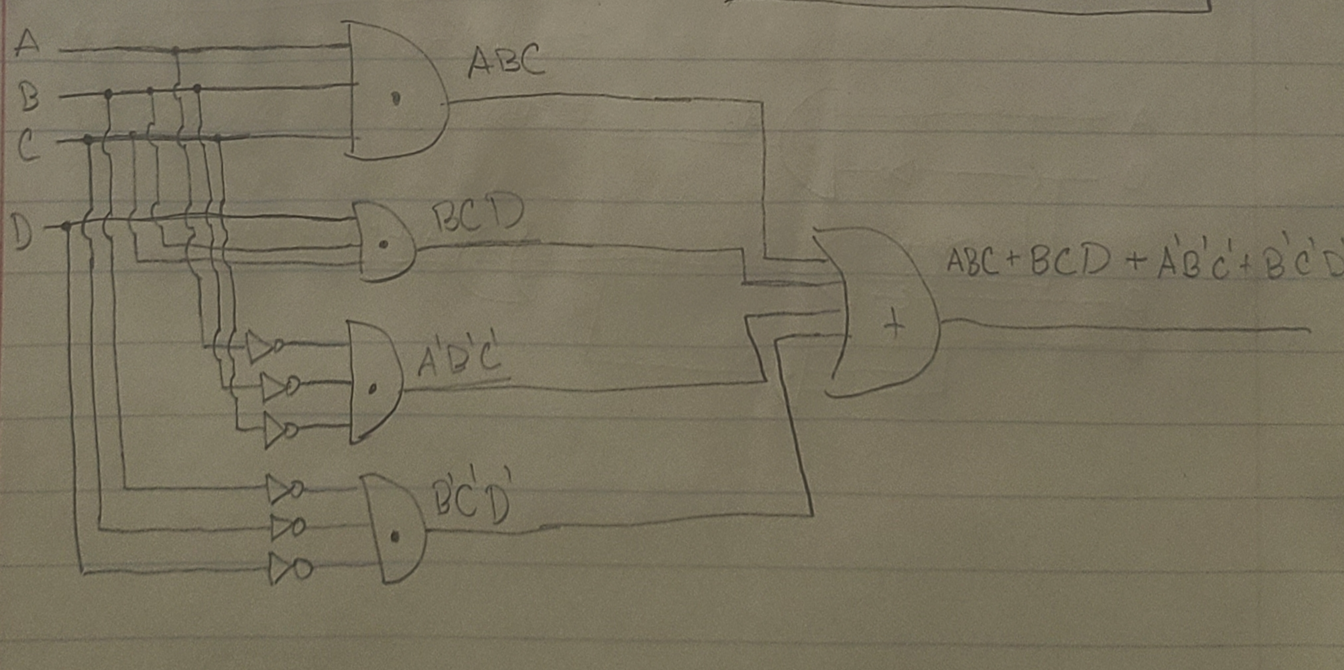
$\underbrace{A'B'C'D'}_{xy'} + \underbrace{A'B'C'D}_{xy} + A'BCD + AB'C'D' + ABCD' + ABCD$

uniting $= A'B'C' + A'BCD + AB'C'D' + \underbrace{ABCD'}_{xy'} + \underbrace{ABCD}_{xy}$

uniting $= A'B'C' + A'BCD + AB'C'D' + ABC$

Dist $= B'C'(A' + AD') + BC(A'D + A)$

Elimination $= \boxed{A'B'C' + B'C'D' + BCD + ABC}$



2) $f(A,B,C,D)$ if $f(A,B,C,D) \geq 5$ & $f(A,B,C,D) \leq 11$ $Z = 1$

else $Z = 0$

A	B	C	D	Z	
0	0	0	0	0	M_0
0	0	0	1	0	M_1
0	0	1	0	0	M_2
0	0	1	1	0	M_3
0	1	0	0	0	M_4
0	1	0	1	1	m_5
0	1	1	0	1	m_6
0	1	1	1	1	m_7
1	0	0	0	1	m_8
1	0	0	1	1	m_9
1	0	1	0	1	m_{10}
1	0	1	1	1	m_{11}
1	1	0	0	0	M_{12}
1	1	0	1	0	M_{13}
1	1	1	0	0	M_{14}
1	1	1	1	0	M_{15}

$\sum m(5,6,7,8,9,10,11)$

$\prod M(0,1,2,3,4,12,13,14,15)$

$m_5 + m_6 + m_7 + m_8 + m_9 + m_{10} + m_{11}$

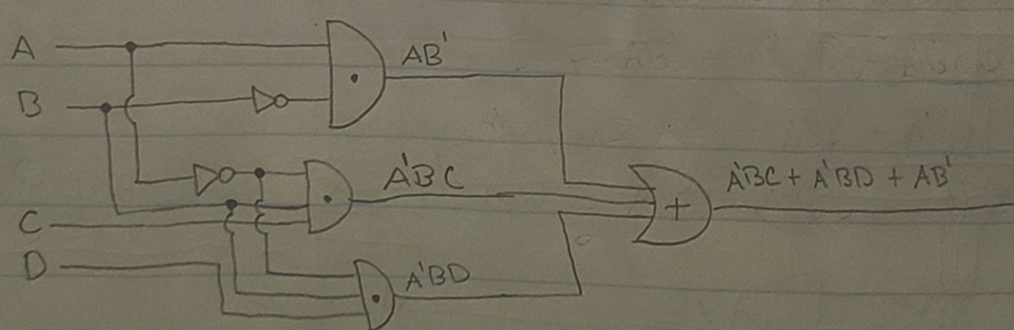
$A'BC'D + A'BCD' + A'BCD + AB'C'D' + AB'C'D + AB'CD' + AB'CD$

Dist $= A'B(C'D + CD' + CD) + AB'(C'D + C'D' + CD' + CD)$

uniting $= A'B(C'D + C) + AB'(C' + C)$

Elimination $= A'B(C + D) + AB'$

Dist $= \boxed{A'BC + A'BD + AB'}$



3) $f(a,b,c) = a(b+c')$

a	b	c	a	b	c'	b+c'	a(b+c')	f'(a,b,c)
0	0	0	0	0	1	1	0	1
0	0	1	0	0	0	0	0	1
0	1	0	0	1	1	1	0	1
0	1	1	0	1	0	1	0	1
1	0	0	1	0	1	1	1	0
1	0	1	1	0	0	0	0	1
1	1	0	1	1	1	1	1	0
1	1	1	1	1	0	1	1	0

$f(a,b,c) = \Sigma m(4,6,7) \rightarrow m_4 + m_6 + m_7$

$\pi M(0,1,2,3,5) \rightarrow M_0 \cdot M_1 \cdot M_2 \cdot M_3 \cdot M_5$

$f'(a,b,c) = \Sigma m(0,1,2,3,5) \rightarrow m_0 + m_1 + m_2 + m_3 + m_5$

$\pi M(4,6,7) \rightarrow M_4 \cdot M_6 \cdot M_7$

4) $f(A,B,C,D) = AB + A'CD$

$\Sigma m(3, 7, 12, 13, 14, 15)$

$m_3 + m_7 + m_{12} + m_{13} + m_{14} + m_{15}$

$A'B'CD + A'BCD + ABC'D' + ABC'D + ABCD' + ABCD$

Dist $= AB(C'D' + C'D + CD' + CD) + A'B'CD + A'BCD$

uniting $= AB(C+C') + A'B'CD + A'BCD$

uniting $= AB + A'C(BD + B'D)$

$= \boxed{AB + A'CD}$

A	B	C	D	AB	A'CD	AB+A'CD	
0	0	0	0	0	0	0	M_0
0	0	0	1	0	0	0	M_1
0	0	1	0	0	0	0	M_2
0	0	1	1	0	1	1	m_3
0	1	0	0	0	0	0	M_4
0	1	0	1	0	0	0	M_5
0	1	1	0	0	0	0	M_6
0	1	1	1	0	1	1	m_7
1	0	0	0	0	0	0	M_8
1	0	0	1	0	0	0	M_9
1	0	1	0	0	0	0	M_{10}
1	0	1	1	0	0	0	M_{11}
1	1	0	0	1	0	1	m_{12}
1	1	0	1	1	0	1	m_{13}
1	1	1	0	1	0	1	m_{14}
1	1	1	1	1	0	1	m_{15}

$\Pi M(0,1,2,4,5,6,8,9,10,11)$

$M_0 \cdot M_1 \cdot M_2 \cdot M_4 \cdot M_5 \cdot M_6 \cdot M_8 \cdot M_9 \cdot M_{10} \cdot M_{11}$

$(A+B+C+D) \cdot (A+B+C+D') \cdot (A+B+C'+D) \cdot (A+B'+C+D) \cdot (A+B'+C+D') \cdot (A+B'+C'+D) \cdot$
$(A'+B+C+D) \cdot (A'+B+C+D') \cdot (A'+B+C'+D) \cdot (A'+B+C'+D')$

Dist $= (A+B)((C+D) \cdot (C+D') \cdot (C'+D)) \cdot (A+B')((C+D) \cdot (C+D') \cdot (C'+D)) \cdot (A'+B)((C+D) \cdot (C+D') \cdot (C'+D) \cdot (C'+D'))$

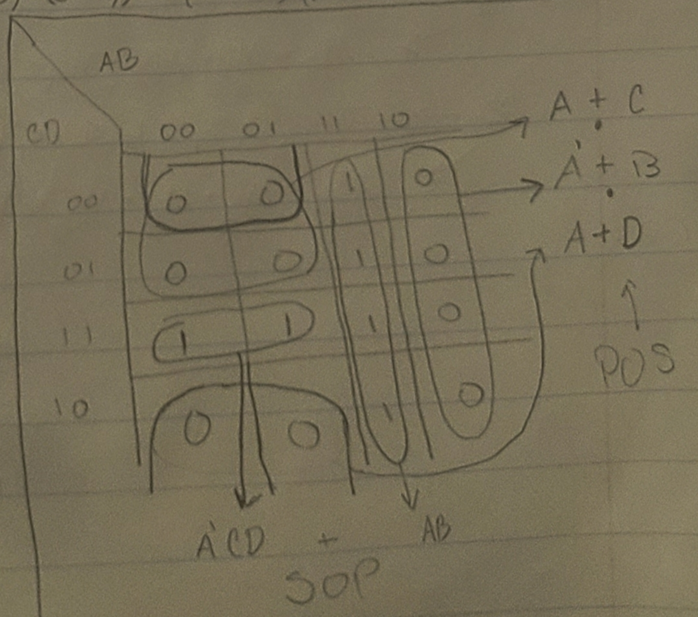
uniting $= (A+B)(D \cdot (C+D')) \cdot (A+B')(C \cdot (C'+D)) \cdot A'+B$

Elimination $= (A+B)(CD) \cdot (A+B')(CD) \cdot A'+B$

uniting $= A(CD) \cdot A'+B$

Dist $= (A+C)(A+D)(A'+B)$

$\boxed{(A+C)(A+D)(A'+B)}$



5) f(A,B,C,D)

A	B	C	D	X Y Z	X		Y		Z		
0	0	0	0	0 0 0	0	M_0	0	M_0	0		M_0
0	0	0	1	0 0 1	0	M_1	0	M_1	1	m_1	
0	0	1	0	0 0 1	0	M_2	0	M_2	1	m_2	
0	0	1	1	0 1 0	0	M_3	1	m_3	0		M_3
0	1	0	0	0 0 1	0	M_4	0	M_4	1	m_4	
0	1	0	1	0 1 0	0	M_5	1	m_5	0		M_5
0	1	1	0	0 1 0	0	M_6	1	m_6	0		M_6
0	1	1	1	0 1 1	0	M_7	1	m_7	1	m_7	
1	0	0	0	0 0 1	0	M_8	0	M_8	1	m_8	
1	0	0	1	0 1 0	0	M_9	1	m_9	0		M_9
1	0	1	0	0 1 0	0	M_{10}	1	m_{10}	0		M_{10}
1	0	1	1	0 1 1	0	M_{11}	1	m_{11}	1	m_{11}	
1	1	0	0	0 1 0	0	M_{12}	1	m_{12}	0		M_{12}
1	1	0	1	0 1 1	0	M_{13}	1	m_{13}	1	m_{13}	
1	1	1	0	0 1 1	0	M_{14}	1	m_{14}	1	m_{14}	
1	1	1	1	1 0 0	1	m_{15}	0	M_{15}	0		M_{15}

a) $X \rightarrow \sum m(15) \rightarrow m_{15}$

$Y \rightarrow \sum m(3,5,6,7,9,10,11,12,13,14)$

$\searrow$ $m_3 + m_5 + m_6 + m_7 + m_9 + m_{10} + m_{11} + m_{12} + m_{13} + m_{14}$

$Z \rightarrow \sum m(1,2,4,7,8,11,13,14)$

$\searrow$ $m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}$

b) $Y \rightarrow \prod M(0,1,2,4,8,15)$

$\searrow$ $M_0 \cdot M_1 \cdot M_2 \cdot M_4 \cdot M_8 \cdot M_{15}$

$Z \rightarrow \prod M(0,3,5,6,9,10,12,15)$

$\searrow$ $M_0 \cdot M_3 \cdot M_5 \cdot M_6 \cdot M_9 \cdot M_{10} \cdot M_{12} \cdot M_{15}$